

Ambiguity in running spectral index with an extra light field during inflation

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At the beginning of inflation there could be extra dynamical scalar fields that will soon disappear (become static) before the end of inflation. In the light of multi-field inflation, those extra degrees of freedom may alter the time-dependence of the original spectrum of the curvature perturbation. It is possible to remove such fields introducing extra number of e-foldings prior to $N_e \sim 60$, however such extra e-foldings may make the trans-Planckian problem worse due to the Lyth bound. We show that such extra scalar fields can change the running of the spectral index to give correction of ± 0.01 without adding significant contribution to the spectral index. The corrections to the spectral index (and the amplitude) could be important in considering global behavior of the corrected spectrum, although they can be neglected in the estimation of the spectrum and its spectral index at the pivot scale. The ambiguity in the running of the spectral index, which could be due to such fields, can be used to nullify tension between BICEP2 and Planck experiments.

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I. INTRODUCTION

The running of the spectral index has sometimes been used to discriminate inflationary models. Recent discovery of B-mode polarization by the BICEP2 collaboration [1] would be an important signal of primordial inflation, since it would indicate the existence of quantum gravitational radiation, which is a generic prediction of inflationary cosmological models [2, 3].¹ Although the tensor perturbation is very powerful in discriminating inflationary models, the BICEP2 result is in some tension with previous experiments such as WMAP [5] and Planck [6], which claimed upper limits on r (tensor to scalar ratio). To resolve the tension, BICEP2 collaboration suggested [1] that the scalar spectral index could run fast. However, the signature of the required running $\alpha_s = -0.022 \pm 0.010$ (68%CL) [1] is negative and its absolute value is very large compared with previous expectation.² In this paper we show that the constraints from the running spectral index could be nullified by introducing an extra free scalar field. Multifield models of inflation may allow inflationary trajectory that is sensitive to the initial condition. Recently, it was pointed out that some multifield models make predictions for the inflationary observables that do not depend strongly on the specific initial condition [8, 9]. On the other hand, observational data could be used to constrain the initial field configuration if the model has sensitivity to the initial condition. Recent numerical calculation [9] clearly shows

that such sensitivity can generate significant running in double quadratic inflation.³ Our result might indicate that the running spectral index could be a free parameter that may not be taken too much seriously in this setup.

II. RUNNING SPECTRAL INDEX WITH A FREE SCALAR FIELD

A. A toy model: Extra component $\dot{\rho}_\chi/\rho_\chi = \text{const.}$

We consider a model in which the curvature perturbation is sourced by the inflaton ϕ . The additional free scalar field is χ , which is dynamical at the beginning of inflation and may contribute to the adiabatic curvature perturbation at that moment.

In this section we introduce extra field χ , which will soon disappear because of $\dot{\rho}_\chi = -6H\rho_\chi < 0$. Note that this toy model has $\dot{\rho}_\chi > 0$, whose contribution to the running is usually $\Delta\alpha_s > 0$. Therefore, although the model is simple and very useful for the intuitive argument, the model is not suitable for generating negative running.

The curvature perturbation at the horizon exit is calculated as

$$\zeta = -H \frac{\delta\rho}{\dot{\rho}}$$

¹ See also Ref. [4], which shows (in contrast to Ref. [1]) that joint BICEP2 + Planck analysis could favor solutions without gravity waves.

² See also recent studies [7]

³ In this paper we investigate a possibility of shifting α_s without changing significantly the other cosmological predictions. Although Fig.3 of Ref.[9] is illuminating, it might be showing that the model could not give $\alpha_s \sim -0.01$ without shifting n_s .

$$\begin{aligned}
&= -H \frac{\delta\rho_\phi + \delta\rho_\chi}{\dot{\rho}_\phi + \dot{\rho}_\chi} \\
&\equiv r_\phi \zeta_\phi + (1 - r_\phi) \zeta_\chi,
\end{aligned} \tag{1}$$

where ρ_i and P_i are the density and the pressure of the component labeled by i . For the inflaton ϕ , they are defined as

$$\begin{aligned}
\rho_\phi &\equiv \frac{1}{2} \left(\dot{\phi} \right)^2 + V(\phi) \\
P_\phi &\equiv \frac{1}{2} \left(\dot{\phi} \right)^2 - V(\phi).
\end{aligned} \tag{2}$$

Here $\rho \equiv \rho_\phi + \rho_\chi$ is defined for the total energy density. We used component perturbation ζ_i and the ratio r_ϕ , which are defined by

$$\begin{aligned}
\zeta_i &\equiv -H \frac{\delta\rho_i}{\dot{\rho}_i}, \\
r_\phi &\equiv \frac{\dot{\rho}_\phi}{\dot{\rho}_\phi + \dot{\rho}_\chi}.
\end{aligned} \tag{3}$$

If χ has a flat potential $V(\chi) = 0$ and initial velocity $v_{\chi 0} \neq 0$, it will have a decaying velocity $\dot{\chi}(t) = v_0 e^{-3Ht}$ and could have a negligible perturbation $\delta\rho_\chi$. The energy density of χ obeys $\dot{\rho}_\chi = -3H(1+w)\rho_\chi$ ($w = 1$). For our calculation we define a ratio between densities: $f \equiv \rho_\chi/\rho_\phi$, for which inflationary expansion requires $f \ll 1$. One might argue that ζ_ϕ is not conserved after horizon exit, since the equation of motion does depend on ρ_χ via the Hubble parameter $H^2 = \frac{\rho}{3M_p^2} = \frac{\rho_\phi + \rho_\chi}{3M_p^2}$. The equation of motion is

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \tag{4}$$

where the subscript with comma means derivatives with respect to the field. Conservation of ζ_ϕ is expected when P_ϕ is practically a unique function of ρ_ϕ [10]. In our scenario, P_ϕ might not be an exact unique function of ρ_ϕ , however for $f \ll 1$, one can expect that both P_ϕ and ρ_ϕ are practically unique functions of $\phi(t)$. Barring small correction from $f \ll 1$, the curvature perturbation after inflation will be $\zeta \simeq \zeta_\phi$ [11], which has the spectrum

$$\begin{aligned}
\mathcal{P}_\zeta &= \left(\frac{H}{\dot{\phi}} \right)^2 \left(\frac{H}{2\pi} \right)^2 \\
&= \frac{1}{8\pi^2 M_p^2} \frac{H^2}{\epsilon_\phi},
\end{aligned} \tag{5}$$

where the slow-roll parameter is defined by

$$\epsilon_\phi \equiv \frac{M_p^2}{2} \left(\frac{V_{,\phi}}{\rho} \right)^2. \tag{6}$$

Here, ζ_ϕ is defined at the horizon exit. For later calculation we also define

$$\eta_\phi \equiv \frac{V_{,\phi\phi}}{3H^2}. \tag{7}$$

We are assuming that the energy density ρ_χ of the additional field is a negligible fraction of the total energy density during inflation. Although ρ_χ is a small fraction of the total energy density, $|\dot{\rho}_\chi| \sim |\dot{\rho}_\phi|$ is possible when ϕ is slow-rolling. (If we make an assumption that our mechanism does not change the original $n_s - 1$, we need to consider $|\dot{\rho}_\chi| \lesssim |\dot{\rho}_\phi|$. In that case, contribution to α_s will be maximum when $|\dot{\rho}_\chi| \sim |\dot{\rho}_\phi|$. We are not fine-tuning the quantities to obtain $|\dot{\rho}_\chi| \sim |\dot{\rho}_\phi|$, since we have little difficulty with obtaining the required α_s .) Moreover, one can easily imagine that $|\ddot{\rho}_\chi| \gg |\ddot{\rho}_\phi|$ could be possible even if $f \ll 1$. Our idea is based on that simple speculation.

Since we are introducing a free scalar field χ , ϵ_H is splitted into two parts:

$$\begin{aligned}
\epsilon_H &\equiv -\frac{\dot{H}}{H^2} \\
&= -\frac{\dot{\rho}_\phi}{6H^3 M_p^2} - \frac{\dot{\rho}_\chi}{6H^3 M_p^2} \\
&\equiv \epsilon_\phi + \epsilon_{H\chi}.
\end{aligned} \tag{8}$$

Introducing $R \equiv \rho_\chi/3H^2 M_p^2$, we find

$$\epsilon_{H\chi} = \frac{3(1+w)}{2} R. \tag{9}$$

Here $\epsilon_{H\chi}$ should be discriminated from ϵ_χ , which will be used in later calculation. $\epsilon_{H\chi}$ is identical to ϵ_χ only when $\ddot{\chi}$ is negligible. Then, using $d \ln k = H dt$, we find for the slow-rolling inflaton:

$$\frac{1}{d \ln k} \epsilon_\phi = 4\epsilon_\phi \epsilon_H - 2\eta_\phi \epsilon_\phi. \tag{10}$$

In the first term, ϵ_H appears instead of ϵ_ϕ , since our definition of ϵ_ϕ uses ρ instead of V . For our calculation we also evaluate

$$\begin{aligned}
\frac{1}{d \ln k} \epsilon_{H\chi} &= \frac{3(1+w)}{2H} \dot{R} \\
&= \frac{3(1+w)}{2H} \left[R \frac{\dot{\rho}_\chi}{\rho_\chi} - 2R \frac{\dot{H}}{H} \right] \\
&= -3(1+w) \epsilon_{H\chi} + 2\epsilon_{H\chi} \epsilon_H.
\end{aligned} \tag{11}$$

The spectral index of the curvature perturbation is

$$\begin{aligned}
n_s - 1 &\equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k} \\
&= -6\epsilon_H + 2\eta_\phi.
\end{aligned} \tag{12}$$

Since $\epsilon_H = \epsilon_\phi + \epsilon_{H\chi}$, the shift caused by the additional field χ is

$$\Delta(n_s - 1) = -6\epsilon_{H\chi}. \tag{13}$$

Since we are considering a conservative scenario in which additional field χ does not change the original index, we

need $6\epsilon_{H\chi} \ll 0.04$. We also have

$$\begin{aligned}
\frac{1}{d \ln k} \eta_\phi &= \frac{1}{H dt} \left[\frac{V_{,\phi\phi}}{3H^2} \right] \\
&= \frac{1}{H} \left(\frac{V_{,\phi\phi\phi} \dot{\phi}}{3H^2} \right) + 2 \left(\frac{V_{,\phi\phi}}{3H^2} \right) \left(-\frac{\dot{H}}{H^2} \right) \\
&= - \left(\frac{V_{,\phi\phi\phi} V_{,\phi}}{9H^4} \right) + 2 \left(\frac{V_{,\phi\phi}}{3H^2} \right) \left(-\frac{\dot{H}}{H^2} \right) \\
&= -\xi_\phi + 2\eta_\phi \epsilon_H,
\end{aligned} \tag{14}$$

where we defined

$$\xi_\phi \equiv \frac{V_{,\phi} V_{,\phi\phi\phi}}{9H^4}. \tag{15}$$

Then, the running of the spectral index is

$$\begin{aligned}
\alpha_s &\equiv \frac{dn_s}{d \ln k} \\
&= -6 [4\epsilon_\phi \epsilon_H - 2\eta_\phi \epsilon_\phi] \\
&\quad + 2 [2\eta_\phi \epsilon_H - \xi_\phi] \\
&\quad - 6 [-3(1+w)\epsilon_{H\chi} + 2\epsilon_{H\chi} \epsilon_H] \\
&\simeq [-24\epsilon_\phi \epsilon_H + 16\eta_\phi \epsilon_\phi - 2\xi_\phi] \\
&\quad + 4\eta_\phi \epsilon_{H\chi} + 18(1+w)\epsilon_{H\chi} - 12\epsilon_{H\chi} \epsilon_H,
\end{aligned} \tag{16}$$

where we can see correction $\Delta\alpha_s \sim 18(1+w)\epsilon_{H\chi} \lesssim 0.12(1+w)$, but the sign of this term is positive. We will see that the correction can be negative when χ is moving on a hilltop potential.

For our scenario, we are considering an inflationary model in which final ζ is determined by ϕ , but an extra component may change scale dependence of the spectrum. Multi-field models of inflation have been studied by many authors [12–14]. From those studies one can see that an additional field, which could be dynamical at the horizon exit but will soon stop or slow down, may not change final ζ at least at the first order. The usual calculation uses adiabatic and entropy perturbations instead of component perturbations, and considers conversion between them. Although in this paper we will consider calculation that is usually considered for the curvature mechanism [11], one will reach the same result using the calculation considered in Ref.[12–14]. A similar idea has been studied when there is no extra field but there is a deviation from the slow-roll velocity [15–20]. In those studies it has been found that the curvature perturbation could not be affected by such additional degrees of freedom if they are disappearing during inflation. Also in Ref. [21–23], it has been pointed out that such field might shift the spectral index and its running. Viewing those studies, we think that the idea of changing the scale dependence of the curvature perturbation using an additional dynamical field could have been known partly, although we could not find explicit calculation that can be used for solving the tension between BICEP2 and other experiments.

B. slow-roll χ

In the above calculation we considered an additional component that obeys $\dot{\rho}_\chi = -3H(1+w)\rho_\chi$. Let us see what happens if χ is a moderately slow-rolling field (but it will soon reach its minimum or will soon be negligible). Using conventional slow-roll parameters for both ϕ and χ , we find the spectral index

$$n_s - 1 = -6\epsilon_H + 2\eta_\phi, \tag{17}$$

where $\epsilon_H \equiv \epsilon_\phi + \epsilon_\chi$. Here we define $\epsilon_\chi \equiv \frac{M_p^2}{2} \frac{V_{,\chi}}{\rho}$, which is equivalent to $\epsilon_{H\chi}$ when $\ddot{\chi}$ is negligible. The running of the spectral index is

$$\begin{aligned}
\alpha_s &= -6 [4\epsilon_\phi \epsilon_H - 2\eta_\phi \epsilon_\phi] \\
&\quad + 2 [2\eta_\phi \epsilon_H - \xi_\phi] \\
&\quad - 6 [4\epsilon_\chi \epsilon_H - 2\eta_\chi \epsilon_\chi] \\
&\simeq [-24\epsilon_\phi \epsilon_H + 16\eta_\phi \epsilon_\phi - 2\xi_\phi] \\
&\quad + 4\eta_\phi \epsilon_\chi - 24\epsilon_\chi \epsilon_H + 12\eta_\chi \epsilon_\chi,
\end{aligned} \tag{18}$$

where the major correction appears in $|12\eta_\chi \epsilon_\chi|$. If we assume $|\Delta(n_s - 1)| = 6\epsilon_\chi < |n_s - 1|$, we will have $|\Delta\alpha_s| < |2\eta_\chi(n_s - 1)| \sim 0.08|\eta_\chi|$. Since η_χ does not appear in the spectral index, there is no bound for $|\eta_\chi|$. Therefore, one can take $|\eta_\chi|$ as large as 0.125, which gives correction as large as $|\Delta\alpha_s| \sim 0.01$. Since we are considering an extra dynamical field that will soon disappear, $\eta_\chi \sim 0.125$ is a conceivable choice for the model. Since $\eta_\chi < 0$ is possible for a hilltop potential, one will be able to find a correction $\Delta\alpha_s \sim -0.01$.

C. fast-roll χ

Alternatively, one can consider a hilltop potential $V(\chi) = -\frac{1}{2}m_\chi^2 \chi^2$ for a fast-rolling field χ and will find [24]

$$\chi(t) = \chi(t_0) e^{Kt}, \tag{19}$$

where

$$K \equiv \frac{3}{2}H \left[1 - \sqrt{1 - \frac{4}{9} \left(\frac{m_\chi^2}{H^2} \right)} \right]. \tag{20}$$

Conventional fast-rolling condition is “ $\epsilon_\chi \ll 1$ but $|\eta_\chi| \sim \mathcal{O}(1)$ ”. Then the slow-roll parameter can be replaced as

$$\begin{aligned}
\epsilon_{H\chi} &= -\frac{\dot{\rho}_\chi}{6H^3 M_p^2} \\
&= \frac{(-K^3 + K m_\chi^2) \chi^2}{6H^3 M_p^2} \\
&= \left(-\frac{K^2}{m_\chi^2} + 1 \right) \frac{K}{H} R,
\end{aligned} \tag{21}$$

where contribution from the kinetic term has not been neglected since $\ddot{\chi}$ could not be negligible for the fast-rolling field. Then we find

$$\begin{aligned} \frac{1}{d \ln k} \epsilon_{H\chi} &= -\frac{\ddot{\rho}_\chi}{6H^4 M_p^2} + 3\epsilon_{H\chi} \epsilon_H \\ &= \frac{2K}{H} \epsilon_{H\chi} + 3\epsilon_{H\chi} \epsilon_H. \end{aligned} \quad (22)$$

The spectral index is

$$n_s - 1 = -6\epsilon_\phi + 2\eta_\phi - 6\epsilon_{H\chi}, \quad (23)$$

which gives $6\epsilon_{H\chi} < 0.04$ for our scenario. The running of the spectral index is

$$\begin{aligned} \alpha_s &= -6[4\epsilon_\phi \epsilon_H - 2\eta_\phi \epsilon_\phi] + 2[2\epsilon_H \eta_\phi - \xi_\phi] \\ &\quad - 6 \left[\frac{2K}{H} \epsilon_{H\chi} + 3\epsilon_{H\chi} \epsilon_H \right] \\ &= [-24\epsilon_\phi \epsilon_H + 16\eta_\phi \epsilon_\phi - 2\xi_\phi] + 4\epsilon_{H\chi} \eta_\phi \\ &\quad - 6 \left[\frac{2K}{H} \epsilon_{H\chi} + 3\epsilon_{H\chi} \epsilon_H \right]. \end{aligned} \quad (24)$$

The correction appears in $-12\epsilon_{H\chi} K/H$, where $\Delta\alpha_s$ is negative for the hilltop potential. If one takes $K/H \sim 1$ (fast-roll), one will find $\Delta\alpha_s \sim -0.01$ for $\epsilon_{H\chi} \sim 8 \times 10^{-4}$.

D. The Curvaton

For the conventional curvaton mechanism, one will find [25]

$$\alpha_s \simeq 2 \frac{\ddot{H}}{H^3} - 4\epsilon_H^2 + 4\epsilon_H \eta_\sigma - 2\xi_\sigma, \quad (25)$$

where σ is the curvaton. If we introduce an extra light field χ , significant correction may appear from the first term, which gives for a fast-rolling χ :

$$\begin{aligned} 2 \frac{\ddot{H}}{H^3} &= \frac{\ddot{\rho}_\chi}{3H^4 M_p^2} + \dots \\ &\sim 2 \left(1 - \frac{K^2}{m_\chi^2} \right) \frac{K^2}{H^2} R. \end{aligned} \quad (26)$$

Therefore, one can easily find similar correction in various other models including the curvaton.

E. A model

In this section we show a specific scenario in which α_s can be shifted by an additional field χ . At the beginning of inflation, the additional field has a hilltop potential

$$V(\chi) = -\frac{1}{2} m_\chi^2 \chi^2, \quad (27)$$

and χ will soon reach the minimum at χ_0 . Here we assume $\chi_0 \sim M_p$. The duration is bounded by $e^{K\Delta t} = \frac{\chi_{\text{end}}}{\chi_{\text{ini}}} < \frac{M_p}{H} \simeq 2.4 \times 10^4$ [1], where $\chi_{\text{ini}} > \delta\chi \simeq H/2\pi$ and $\chi_{\text{end}} < \chi_0 \sim M_p$ has been considered. We find $\Delta N \lesssim 10 \times \left(\frac{H}{K}\right)$. The additional field χ must reach its minimum before the end of inflation.

For a moderately slow-rolling χ , ϵ_χ is determined by m_χ and H . Then $|\Delta(n_s - 1)| < 0.04$ gives for the quadratic potential:

$$\begin{aligned} 6\epsilon_\chi &= 6 \times \frac{M_p^2}{2} \left(\frac{-m_\chi^2 \chi}{\rho} \right)_*^2 = 6 \times \left(\frac{-m_\chi^2}{3H^2} \right)_* \left(\frac{-\frac{1}{2} m_\chi^2 \chi^2}{\rho} \right)_* \\ &= 6\eta_\chi R_* < 0.04. \end{aligned} \quad (28)$$

Here the quantities with star is defined when the perturbation leaves horizon. We consider $\eta_\chi \sim 0.6$ and $|R_*| \sim 0.01$ on the scale where significant α_s could be observed. The condition $|R_*| \sim 0.01$ is conceivable since it is required at the very early stage of inflation. Then, we can estimate the correction as

$$|\Delta\alpha_s| \simeq |12\epsilon_\chi \eta_\chi| = |12\eta_\chi^2 R| \sim 0.04. \quad (29)$$

In the same way, the running of α_s will be shifted by a similar order, which is consistent with Planck experiment.⁴

On the other hand, one might suspect that higher runnings may overcome the correction coming from α_s . The expansion will be [26]

$$\begin{aligned} \mathcal{P}_\zeta(k) &= \mathcal{P}_\zeta(k_0) \exp \left[(n_s - 1) \ln \left(\frac{k}{k_0} \right) + \frac{\alpha_s}{2} \ln^2 \left(\frac{k}{k_0} \right) \right. \\ &\quad \left. + \frac{\beta_s}{3!} \ln^3 \left(\frac{k}{k_0} \right) + \dots \right] \\ &= \mathcal{P}_\zeta(k_0) \left(\frac{k}{k_0} \right)^{(n_s-1) + \frac{\alpha_s}{2} \ln \left(\frac{k}{k_0} \right) + \frac{\beta_s}{3!} \ln^2 \left(\frac{k}{k_0} \right) + \dots} \end{aligned} \quad (30)$$

Note that there will be no problem in the expansion if one takes $\ln(k/k_0) = \ln(0.05/0.0027) \simeq 2.92$ and $\beta_s < \alpha_s$, where β_s is the running of α_s . Our model suggests that α_s and β_s could be a similar order. However, normally β_s does not exceed α_s , and the higher runnings does not spoil the scenario of nullifying the tension between BICEP2 and the Planck experiments.

We are anticipating that there could be some significant sign of new physics in the higher runnings, however at this moment higher runnings are not so much constrained. Such a large running (and a running of running) will be checked by the future B-mode polarization and the 21cm line observations. [26]. One may use this to distinguish the source of the scale dependence if a signature might be observed in future experiments.

⁴ See Fig.3 of Ref. [6].

III. CONCLUSION AND DISCUSSION

Normally in single-field inflation models, we have only one scalar field that is responsible for both the inflationary expansion and the production of the curvature perturbation. In reality, however, there could be many other scalar fields *besides* the inflaton field, which could be dynamical at least at the beginning of the inflationary stage. Although the non-inflaton components may disappear (or becomes static) before the end of inflation, those additional fields could change the scale dependence of the spectrum. If one wants to remove those extra fields before the onset of $N_e \sim 60$ inflation, one can assume extra e-foldings prior to $N_e \sim 60$, so that such fast-rolling (or modestly fast-rolling) fields are already negligible from the beginning. In that case, remaining fields will not have significant contribution to α_s . On the other hand, in the light of BICEP2 and the Lyth bound, such extra e-foldings would require more excursion of the inflaton field, which could be a problem if one takes the trans-Planckian problem seriously. We thus expect that such extra dynamical degrees of freedom can exist naturally in modern inflationary scenarios. As long as we know, the idea has not been discussed yet to solve the tension between BICEP2 and other experiments.

In this paper we considered a free scalar field and calculated its contribution to n_s and α_s . We assumed conservatively that spectral index is not much altered by the additional field. Using a simple model, we showed that $O(0.01)$ ambiguity can be found in the running spectral

⁵ The second order (i.e., the running of running of r) can be large since it will include $\ddot{\epsilon}_\phi$.

⁶ We thank the reviewer of the journal for these points.

index if an extra field could be dynamical when the perturbation leaves horizon. In the same way, the running of α_s can be shifted by a similar order, which could be used to distinguish the source of the scale dependence. At the same time, the scale-dependence of the tensor mode could be affected, which has already been discussed in Ref.[27–29]. In our scenario, we find $r \sim 16\epsilon_\phi$, which does not include ϵ_χ . This means that r is not affected by $\epsilon_\chi \neq 0$. Similarly, the running of r is determined by $\dot{\epsilon}_\phi$, which does not introduce significant running at the first order. Significant running may appear from the next order of the running.⁵ On the other hand, in our model $\frac{dn_t}{d \ln k} \propto \dot{\epsilon}_H$ can be large. Since the scale-dependence of the tensor power spectrum is significant in our case, while such a scale dependence is usually not assumed, there could be concern that it might bias the estimation of parameters like r and α_s from the WMAP/Planck data. Those quantities could be negligible at the end, but the resolution of the tension between BICEP2 and the WMAP/Planck could be affected. In that sense we need further investigation to obtain more accurate estimation of the parameters when the tensor mode is running.⁶

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